5.4 Solving Percent Problems Using the Percent Equation

In this section we will develop and use a more algebraic equation approach to solving percent equations. Recall the percent proportion from the last section:

\[
\frac{\text{percent}}{100} = \frac{\text{percentage}}{\text{base}}
\]

Considering that \(\frac{\text{percent}}{100}\) is merely a percent divided by 100, we can consider the decimal form of it in the equation:

\[
\text{percent} = \frac{\text{percentage}}{\text{base}}
\]

\[
\text{base} \cdot \text{percent} = \text{base} \cdot \left(\frac{\text{percentage}}{\text{base}}\right)
\]

\[
\text{percentage} = \text{percent} \cdot \text{base}
\]

This last statement is called the **percent equation**. Remember that the term percent refers to the decimal (or fraction) form, which has already been divided by 100. This percent equation involves the familiar quantities from the past section: percent, base, and percentage. Once identified, the method of solving the equation utilizes our steps for solving equations from Chapter 4. Suppose we are asked to find 18% of 250. The percent is 18% = 0.18 (converted to a decimal), and the base is 250. Using the percent equation:

\[
\text{percentage} = \text{percent} \cdot \text{base}
\]

\[
x = 0.18 \cdot 250
\]

\[
x = 45
\]

Thus 18% of 250 is 45. Let’s use this approach to re-solve Example 1 from the last section.

**Example 1**  Find the following percentages.

- **a.** 48% of 120
- **b.** 45% of 286
- **c.** 7.25% of 1500
- **d.** \(15 \frac{1}{2}\)% of 2400
- **e.** 150% of 260
Solution

a. The percent is 48% = 0.48 (converted to decimal) and the base is 120. Using the percent equation:
   \[ \text{percentage} = \text{percent} \times \text{base} \]
   \[ x = 0.48 \times 120 \]
   \[ x = 57.6 \]
   Thus 48% of 120 is 57.6.

b. The percent is 45% = 0.45 (converted to decimal) and the base is 286. Using the percent equation:
   \[ \text{percentage} = \text{percent} \times \text{base} \]
   \[ x = 0.45 \times 286 \]
   \[ x = 128.7 \]
   Thus 45% of 286 is 128.7.

c. The percent is 7.25% = 0.0725 (converted to decimal) and the base is 1500. Using the percent equation:
   \[ \text{percentage} = \text{percent} \times \text{base} \]
   \[ x = 0.0725 \times 1500 \]
   \[ x = 108.75 \]
   Thus 7.25% of 1500 is 108.75.

d. The percent is \(15\frac{1}{2}\)% = 15.5% = 0.155 (converted to decimal) and the base is 2400. Using the percent equation:
   \[ \text{percentage} = \text{percent} \times \text{base} \]
   \[ x = 0.155 \times 2400 \]
   \[ x = 372 \]
   Thus \(15\frac{1}{2}\)% of 2400 is 372.

e. The percent is 150% = 1.50 (converted to decimal) and the base is 260. Using the percent equation:
   \[ \text{percentage} = \text{percent} \times \text{base} \]
   \[ x = 1.50 \times 260 \]
   \[ x = 390 \]
   Thus 150% of 260 is 390.
The second type of percent problem involved one in which the percentage is given and the base is unknown. Suppose we know that 68% of a number is 204. The percent is 68% = 0.68, converted to a decimal. This time the percentage is 204, and the base is unknown. Using the percent equation:

\[ \text{percent} \cdot \text{base} = \text{percentage} \]
\[ 0.68x = 204 \]
\[ x = \frac{204}{0.68} = 300 \]

Thus 68% of 300 is 204. Note that the percent equation provides an easy check of our answer, since 0.68 \( \times \) 300 = 204, verifying our answer. As more practice, let’s re-solve Example 2 from the last section.

**Example 2** Solve the following percent problems.

a. 24% of what number is equal to 120?

b. 35% of what number is equal to 51.8?

c. 7.5% of what number is equal to 270?

d. \( \frac{1}{4} \)% of what number is equal to 275?

e. 125% of what number is equal to 800?

**Solution**

a. The percent is 24% = 0.24 (converted to decimal) and the percentage is 120. Using the percent equation:

\[ \text{percent} \cdot \text{base} = \text{percentage} \]
\[ 0.24x = 120 \]
\[ x = \frac{120}{0.24} = 500 \]

Thus 24% of 500 is 120. Checking the value: 0.24 \( \times \) 500 = 120.

b. The percent is 35% = 0.35 (converted to decimal) and the percentage is 51.8. Using the percent equation:

\[ \text{percent} \cdot \text{base} = \text{percentage} \]
\[ 0.35x = 51.8 \]
\[ x = \frac{51.8}{0.35} = 148 \]

Thus 35% of 148 is 51.8. Checking the value: 0.35 \( \times \) 148 = 51.8.
c. The percent is 7.5% = 0.075 (converted to decimal) and the percentage is 270. Using the percent equation:
   \[ \text{percent} \times \text{base} = \text{percentage} \]
   \[ 0.075x = 270 \]
   \[ x = \frac{270}{0.075} = 3600 \]
   Thus 7.5% of 3600 is 270. Checking the value: 0.075 \times 3600 = 270.

d. The percent is \( \frac{1}{4} \) = 6.25% = 0.0625 (converted to decimal) and the percentage is 275. Using the percent equation:
   \[ \text{percent} \times \text{base} = \text{percentage} \]
   \[ 0.0625x = 275 \]
   \[ x = \frac{275}{0.0625} = 4400 \]
   Thus \( \frac{1}{4} \)% of 4400 is 275. Checking the value: 0.0625 \times 4400 = 275.

e. The percent is 125% = 1.25 (converted to decimal) and the percentage is 800. Using the percent equation:
   \[ \text{percent} \times \text{base} = \text{percentage} \]
   \[ 1.25x = 800 \]
   \[ x = \frac{800}{1.25} = 640 \]
   Thus 125% of 640 is 800. Checking the value: 1.25 \times 640 = 800.

The third type of percent problem is where both the percentage and base are given, but the percent is unknown. Suppose we want to know what percent 208 represents out of 320. The percentage is 208, the base is 320, and the percent is unknown. Using the percent equation:

   \[ \text{percent} \times \text{base} = \text{percentage} \]
   \[ p \times 320 = 208 \]
   \[ p = \frac{208}{320} = 0.65 = 65\% \]
Thus 208 represents 65% of 320. Again note that we can check the equation: \(0.65 \times 320 = 208\). Note the use of the variable \(p\) in this equation. It was used as a reminder that \(p\) represents a percent, and thus must be converted back to a percent for the final step. This is a good habit to get into in solving percent equations. Now let’s re-solve Example 3 from the last section.

**Example 3** Find the following percents.

a. 36 is what percent of 80?

b. 132 is what percent of 165?

c. 120 is what percent of 180?

d. 200 is what percent of 160?

**Solution**

a. The percentage is 36, the base is 80, and the percent is unknown. Using the percent equation:

\[
\text{percent} \times \text{base} = \text{percentage}
\]

\[
p \times 80 = 36
\]

\[
p = \frac{36}{80}
\]

\[
p = 0.45 = 45\%
\]

Thus 36 is 45% of 80. Checking the value: \(0.45 \times 80 = 36\).

b. The percentage is 132, the base is 165, and the percent is unknown. Using the percent equation:

\[
\text{percent} \times \text{base} = \text{percentage}
\]

\[
p \times 165 = 132
\]

\[
p = \frac{132}{165}
\]

\[
p = 0.80 = 80\%
\]

Thus 132 is 80% of 165. Checking the value: \(0.80 \times 165 = 132\).

c. The percentage is 120, the base is 180, and the percent is unknown. Using the percent equation:

\[
\text{percent} \times \text{base} = \text{percentage}
\]

\[
p \times 180 = 120
\]

\[
p = \frac{120}{180}
\]

\[
p = \frac{2}{3} = 66\frac{2}{3}\%
\]
Thus 120 is $\frac{2}{3}$% of 180. Note in this problem we did not convert to a decimal, since a repeating decimal would have resulted.

Checking the value: $\frac{2}{3} \times 180 = 120$.

d. The percentage is 200, the base is 160, and the percent is unknown. Using the percent equation:

$$p \times 160 = 200$$

$$p = \frac{200}{160}$$

$$p = 1.25 = 125\%$$

Thus 200 is 125% of 160. Checking the value: $1.25 \times 160 = 200$.

Often students find the percent equation easier to use than the proportion method. Keep in mind that both methods require you to recognize the percent, the base, and the percentage. We will re-solve Example 4 from the previous section, however this time we use the percent equation approach, rather than percent proportions.

**Example 4** Solve the following percent problems.

a. $4 \frac{1}{2}$% of 620 is what number?

b. 35% of what number is 30.1?

c. 112 is what percent of 140?

d. 297 represents 45% of what number?

e. 19.8% of 1200 is what number?

f. 240 represents what percent of 180?
Solution

a. The percent, base, and percentage are:

percent = \(4 \frac{1}{2}\% = 4.5\% = 0.045\)

base = 620

percentage = \(x\) (unknown)

Using the percent equation:

\[ \text{percentage} = \text{percent} \times \text{base} \]

\[ x = 0.045 \times 620 \]

\[ x = 27.9 \]

Thus \(4 \frac{1}{2}\%\) of 620 is 27.9.

b. The percent, base, and percentage are:

percent = 35\% = 0.35

base = \(x\) (unknown)

percentage = 30.1

Using the percent equation:

\[ \text{percent} \times \text{base} = \text{percentage} \]

\[ 0.35x = 30.1 \]

\[ x = \frac{30.1}{0.35} = 86 \]

Thus 35\% of 86 is 30.1. Checking the value: 0.35 \(\times\) 86 = 30.1.

c. The percent, base, and percentage are:

percent = \(p\) (unknown)

base = 140

percentage = 112

Using the percent equation:

\[ \text{percent} \times \text{base} = \text{percentage} \]

\[ p \times 140 = 112 \]

\[ p = \frac{112}{140} \]

\[ p = 0.80 = 80\% \]

Thus 112 is 80\% of 140. Checking the value: 0.80 \(\times\) 140 = 112.
d. The percent, base, and percentage are:
   percent = 45% = 0.45
   base = x (unknown)
   percentage = 297
Using the percent equation:
   percent \• base = percentage
   0.45x = 297
   \[ x = \frac{297}{0.45} = 660 \]
   Thus 297 represents 45% of 660. Checking the value: 0.45 \• 660 = 297.

e. The percent, base, and percentage are:
   percent = 19.8% = 0.198
   base = 1200
   percentage = x (unknown)
Using the percent equation:
   percentage = percent \• base
   x = 0.198 \• 1200
   \[ x = 237.6 \]
   Thus 19.8% of 1200 is 237.6.

f. The percent, base, and percentage are:
   percent = p (unknown)
   base = 180
   percentage = 240
Using the percent equation:
   percent \• base = percentage
   p \• 180 = 240
   \[ p = \frac{240}{180} \]
   \[ p = \frac{4}{3} = 133\frac{1}{3}\% \]
   Thus 240 represents 133\(\frac{1}{3}\)% of 180. Checking the value: \(\frac{4}{3}\) \• 180 = 240
Note again we used fractions here to avoid using repeating decimals.
Terminology

percent equation

Exercise Set 5.4

Solve the following percent problems. Remember that you will need to recognize the percent, the base, and the percentage in order to set up the percent equation.

1. 69% of 400 is what number?
2. 65% of 148 is what number?
3. 35% of what number is 280?
4. 55% of what number is 275?
5. 136 is what percent of 160?
6. 288 is what percent of 400?
7. 37% of 1250 is what number?
8. 56% of 1420 is what number?
9. 544 is what percent of 640?
10. 195 is what percent of 250?
11. 78% of what number is 1029.6?
12. 57% of what number is 826.5?
13. 8.75% of 24,000 is what number?
14. 8.3% of 1,300 is what number?
15. 47.5% of what number is 237.5?
16. 52.8% of what number is 316.8?
17. 316 is what percent of 474?
18. 750 is what percent of 900?
19. $15\frac{1}{3}$% of 900 is what number?
20. $21\frac{3}{4}$% of 800 is what number?
21. 140% of what number is 91?
22. 160% of what number is 83.2?
23. 210 is what percent of 150?
24. 500 is what percent of 360?
25. $15\frac{2}{3}$% of 2100 is what number?
26. $5\frac{3}{4}$% of 840 is what number?
27. $8\frac{1}{2}$% of what number is 8.079?
28. $10\frac{3}{4}$% of what number is 90.3?
29. 36 is what percent of 20?
30. 450 is what percent of 250?
31. 285% of 340 is what number?
32. 228% of 450 is what number?
33. $8\frac{1}{3}$% of what number is 100?
34. $9\frac{2}{3}$% of what number is 147.9?
35. 413.1 is what percent of 486?
36. 482.4 is what percent of 720?
37. 625% of 80 is what number?
38. 500% of 64 is what number?
39. 320% of what number is 240?
40. 420% of what number is 441?