Chapter 5
Ratio, Proportion, and Percent

5.1 Ratios and Proportions

In this section we extend the idea of equivalent fractions, converting fractions to decimals, and equations. Recall that two fractions are equivalent (or equal) if they both have the same decimal form. For example, the fractions $\frac{3}{4}$ and $\frac{12}{16}$ are equivalent, since they both have a decimal form of 0.75. Note that they are also equivalent since $\frac{12}{16}$ can be reduced to $\frac{3}{4}$, but this connection is not as relevant to our discussions in this section. There are other quantities which are equivalent to 0.75, such as $\frac{4.5}{6}$ and $\frac{1.25}{2}$, though they generally do not fit the form of a simplified fraction. They do fit the form of a ratio, which refers to any quotient of two quantities. Ratios are often associated with units to give the quantities meaning. The following are examples of these types of ratios:

- speed of a car: $\frac{130 \text{ miles}}{2 \text{ hours}} = 65 \text{ miles per hour}$
- unit cost of food: $\frac{47 \text{ cents}}{2.5 \text{ ounces}} = 18.8 \text{ cents per ounce}$
- cost of lumber: $\frac{8.50 \text{ dollars}}{4 \text{ feet}} = 2.125 \text{ dollars per foot}$

Note the use of the word “per”, which is frequently used in expressing ratios.

Example 1 Write the following mathematical quantities as a ratio, and simplify.

a. 214 miles traveled in 3 hours
b. $17.10 paid for 3.6 pounds of steak
c. $74.50 paid for 24 pieces of 2x4 lumber
d. $195,000 paid for 15.6 acres of land
Solution

a. Write the quantity as a ratio, then compute the quotient:
\[
\frac{214 \text{ miles}}{3 \text{ hours}} = 71\frac{1}{3} \text{ miles per hour}
\]

b. Write the quantity as a ratio, then compute the quotient:
\[
\frac{17.10 \text{ dollars}}{3.6 \text{ pounds}} = 4.75 \text{ dollars per pound}
\]

c. Write the quantity as a ratio, then compute the quotient:
\[
\frac{74.50 \text{ dollars}}{24 \text{ pieces}} \approx 3.10 \text{ dollars per piece}
\]
Note that we rounded to the nearest cent, since the quotient results in a repeating decimal. The use of the symbol \( \approx \) indicates rounding of the number.

d. Write the quantity as a ratio, then compute the quotient:
\[
\frac{195,000 \text{ dollars}}{15.6 \text{ acres}} = 12,500 \text{ dollars per acre}
\]

Ratios are useful in comparing quantities, especially prices. Consider the following two options for purchasing canned peaches:

- 8 ounces for $1.49
- 35 ounces for $6.39

Assuming you need a large quantity of canned peaches, which purchase offers the best value? We will compute the unit value for each, which is the price per ounce. Computing the ratios:
\[
\frac{1.49 \text{ dollars}}{8 \text{ ounces}} = 0.1863 \text{ dollars per ounce} \approx 18.63 \text{ cents per ounce}
\]
\[
\frac{6.39 \text{ dollars}}{35 \text{ ounces}} = 0.1826 \text{ dollars per ounce} \approx 18.26 \text{ cents per ounce}
\]
The larger quantity of peaches offers the best value, due to a lower unit price. The concept of unit price can be applied to other ratios, such as speed of cars (miles per hour) and production of a factory (units per day).
Example 2  Compare the two given quantities using unit value comparisons.

a. Orange Juice
   
   8 oz for $1.19
   20 oz for $2.79

b. Speed of a Car
   
   170 miles in 2.5 hours
   225 miles in 3.2 hours

c. Price of Land
   
   $56,000 for 1.75 acres
   $86,800 for 3.1 acres

d. Price per Earnings of Stock
   
   $94.71 for $3.85 of company earnings
   $48.26 for $1.86 of company earnings

Solution  

a. Computing the price per ounce ratios:

\[
\frac{1.19 \text{ dollars}}{8 \text{ ounces}} = 0.1488 \text{ dollars per ounce} \approx 14.88 \text{ cents per ounce}
\]

\[
\frac{2.79 \text{ dollars}}{20 \text{ ounces}} = 0.1395 \text{ dollars per ounce} \approx 13.95 \text{ cents per ounce}
\]

The larger quantity (20 ounces) offers the lower price per ounce.

b. Computing the speed (miles per hour) of each car:

\[
\frac{170 \text{ miles}}{2.5 \text{ hours}} = 68 \text{ miles per hour}
\]

\[
\frac{225 \text{ miles}}{3.2 \text{ hours}} = 70.3 \text{ miles per hour}
\]

The shorter distance (170 miles) offers the slower speed of the car.

c. Computing the price per acre of land:

\[
\frac{56000 \text{ dollars}}{1.75 \text{ acres}} = 32,000 \text{ dollars per acre}
\]

\[
\frac{86800 \text{ dollars}}{3.1 \text{ acres}} = 28,000 \text{ dollars per acre}
\]

The larger parcel (3.1 acres) offers the lower price per acre.
d. Computing the price per dollar of company earnings:

\[
\frac{94.71 \text{ dollars}}{3.85 \text{ dollars earnings}} = 24.60 \text{ dollars per dollar earnings}
\]

\[
\frac{48.26 \text{ dollars}}{1.86 \text{ dollars earnings}} = 25.95 \text{ dollars per dollar earnings}
\]

The more expensive stock ($94.71) has a lower price per earnings ratio.

Up to this point we have constructed ratios for the purpose of comparing them. Suppose, however, that we know two ratios are equal. Any time two ratios are equal we call the resulting equation a proportion. That is, a proportion is a statement of the form:

\[
\frac{a}{b} = \frac{c}{d}
\]

To solve a proportion, we multiply by the LCM = \(bd\):

\[
bd \cdot \frac{a}{b} = bd \cdot \frac{c}{d}
\]

\[
ad = bc
\]

Let’s look at an example. Suppose we are given the proportion:

\[
\frac{x}{4} = \frac{9}{20}
\]

Multiply by the LCM = 20, then solve the resulting equation:

\[
\frac{x}{4} = \frac{9}{20}
\]

\[
20 \cdot \frac{x}{4} = 20 \cdot \frac{9}{20}
\]

\[
5x = 9
\]

\[
\frac{1}{5} \cdot 5x = \frac{1}{5} \cdot 9
\]

\[
x = \frac{9}{5}
\]
The following example will illustrate how to solve proportions with the variable in different locations.

Example 3 Solve each proportion.

a. \(\frac{x}{-6} = \frac{5}{18}\)

b. \(\frac{5}{y} = \frac{4}{6.2}\)

c. \(-\frac{4}{13} = \frac{a}{5}\)

d. \(-\frac{5}{8} = \frac{6}{b}\)

e. \(\frac{x + 2}{3} = \frac{x - 4}{2}\)

Solution

a. Multiplying by the LCM = 18:

\[
x = \frac{5}{18}
\]

\[
x \cdot \frac{18}{-6} = \frac{5}{18} \cdot \frac{18}{18}
\]

\[
-3x = 5
\]

\[
-\frac{3}{1} \cdot (-3x) = -\frac{3}{1} \cdot 5
\]

\[
x = -\frac{5}{3}
\]
b. Multiplying by the LCM = 6.2y:
\[
\frac{5}{y} = \frac{4}{6.2}
\]
\[
6.2y \cdot \frac{5}{y} = 6.2y \cdot \frac{4}{6.2}
\]
\[
31 = 4y
\]
\[
\frac{1}{4} \cdot 4y = \frac{1}{4} \cdot 31
\]
\[
y = \frac{31}{4} = 7.75
\]

c. Multiplying by the LCM = 65:
\[
-\frac{4}{13} = \frac{a}{5}
\]
\[
65 \cdot \left( -\frac{4}{13} \right) = 65 \cdot \frac{a}{5}
\]
\[
-20 = 13a
\]
\[
\frac{1}{13} \cdot 13a = \frac{1}{13} \cdot (-20)
\]
\[
a = -\frac{20}{13}
\]

d. Multiplying by the LCM = 8b:
\[
-\frac{5}{8} = \frac{6}{b}
\]
\[
8b \cdot \left( -\frac{5}{8} \right) = 8b \cdot \frac{6}{b}
\]
\[
-5b = 48
\]
\[
-\frac{1}{5} \cdot (-5b) = -\frac{1}{5} \cdot (48)
\]
\[
b = -\frac{48}{5}
\]
e. Multiplying by the LCM = 6:

\[ \frac{x + 2}{3} = \frac{x - 4}{2} \]

\[ 6 \cdot \frac{x + 2}{3} = 6 \cdot \frac{x - 4}{2} \]

\[ 2(x + 2) = 3(x - 4) \]

\[ 2x + 4 = 3x - 12 \]

\[ 2x - 3x + 4 = 3x - 3x - 12 \]

\[ -x + 4 = -12 \]

\[ -x + 4 - 4 = -12 - 4 \]

\[ -x = -16 \]

\[ -1 \cdot (-x) = -1 \cdot (-16) \]

\[ x = 16 \]

There are a wide variety of applications which can be solved using proportions. One consumer applications is that of taxes, in which property is taxed as a proportion of its value.

**Example 4**  In one state, property is taxed at a rate of $1.04 for every $120 in property value. If a home has a property value of $180,000, how much will be the property tax?

**Solution**  We look at the ratio of \( \frac{\text{tax dollars}}{\text{property value}} \). This ratio is the same with the state ratio value as with the actual property value. Let \( T \) represent the tax on the property. Solving the proportion:

\[ \frac{T}{180,000} = \frac{1.04 \text{ tax}}{120 \text{ property value}} \]

\[ 180000 \cdot \frac{T}{180000} = 180000 \cdot \frac{1.04}{120} \]

\[ T = 1560 \]

The property tax will be $1560.
Example 5 A map uses a scale of \( \frac{1}{4} \) inch = 30 miles. On a map, two cities are \( 3\frac{1}{8} \) inches apart. How many miles apart are the two cities?

Solution We look at the ratio of \( \frac{\text{inches (map)}}{\text{miles (actual)}} \). This ratio is the same with the map scale as with the actual cities. Let \( d \) represent the distance the two cities are apart. Solving the proportion:

\[
\frac{3\frac{1}{8} \text{ inches}}{d} = \frac{\frac{1}{4} \text{ inch}}{30 \text{ miles}}
\]

\[
30d \cdot \frac{3\frac{1}{8}}{d} = 30d \cdot \frac{1}{30}
\]

\[
30 \cdot 3\frac{1}{8} = \frac{1}{4}d
\]

\[
\frac{1}{4}d = 30 \cdot \frac{25}{8}
\]

\[
\frac{1}{4}d = \frac{375}{4}
\]

\[
4 \cdot \frac{1}{4}d = 4 \cdot \frac{375}{4}
\]

\[
d = 375
\]

The two cities are 375 miles apart.

A variety of other applications of proportions will be explored in the exercises of this section, as well as throughout this entire chapter.

Terminology

- **ratio**
- **unit value**
- **proportion**
Exercise Set 5.1

Write the following mathematical quantities as a ratio, and simplify.

1. (driving) 335 miles traveled in 5 hours
2. (driving) 288 miles traveled in 4 hours
3. (driving) 357 miles traveled in 6 hours
4. (driving) 508 miles traveled in 8 hours
5. (consumer) $5.16 paid for 2.15 pounds of mushrooms
6. (consumer) $30.75 paid for 8.2 pounds of baby back ribs
7. (consumer) $17.92 paid for 3.2 pounds of filet mignon
8. (consumer) $32.64 paid for 13.6 pounds of potatoes
9. (construction) $948 paid for 8 doors
10. (construction) $576.75 paid for 15 solid beams
11. (construction) $3,187.50 paid for 250 yards of carpet
12. (construction) $5,850 paid for 450 feet of fencing
13. (land value) $75,600 paid for \(\frac{3}{4}\) acres of land
14. (land value) $257,600 paid for 184 acres of land
15. (land value) $63,000 paid for \(\frac{2}{3}\) acres of land
16. (land value) $34,000 paid for \(\frac{2}{3}\) acres of land

Compare the two given quantities using unit value comparisons.

17. Orange Juice
   12 oz for $1.36
   17 oz for $1.95
18. Orange Juice
   8 oz for $1.10
   15 oz for $2.05
19. Toilet Paper
   8 rolls for $2.88
   15 rolls for $5.40
20. Toilet Paper
   12 rolls for $3.84
   20 rolls for $5.80
21. Speed of a Car
   235 miles in 4.2 hours
   295 miles in 5.6 hours
22. Speed of a Car
   167 miles in 2.7 hours
   246 miles in 4 hours
23. Speed of a Boat
   38 nautical miles in 3 hours
   92 nautical miles in 8 hours
24. Speed of a Boat
   204 nautical miles in 24 hours
   246 nautical miles in 30 hours
25. Price of Land
   $43,200 for 1.8 acres
   $98,700 for 4.2 acres

26. Price of Land
   $19,500 for \( \frac{3}{4} \) acre
   $18,000 for \( \frac{2}{3} \) acre

27. Price of Land
   $36,000 for \( 1 \frac{1}{3} \) acres
   $92,750 for \( 3 \frac{1}{2} \) acres

28. Price of Land
   $33,600 for 0.64 acres
   $26,936 for 0.52 acres

29. Gas Mileage
   322 miles on 18 gallons of gas
   356 miles on 21.6 gallons of gas

30. Gas Mileage
   310 miles on 9.7 gallons of gas
   365 miles on 11.8 gallons of gas

31. Boating Slip Fees
   $283.50 for a 42 foot motor yacht
   $331.50 for a 51 foot ketch

32. Boating Slip Fees
   $172.50 for a 30 foot sloop
   $221.40 for a 36 foot cutter

33. Price per Earnings of Stock
   $148.60 for $4.56 of company earnings
   $163.80 for $5.20 of company earnings

34. Price per Earnings of Stock
   $35.10 for $2.60 of company earnings
   $83.20 for $6.50 of company earnings

35. Pitcher’s Earned Run Average (baseball)
   4.2 earned runs for 10 innings pitched
   10 earned runs for 22 innings pitched

36. Pitcher’s Earned Run Average (baseball)
   98 earned runs for 254 innings pitched
   106 earned runs for 275 innings pitched

37. Point Guard Turnover to Assist Ratio (basketball)
   45 turnovers for 156 assists
   175 turnovers for 928 assists

38. Point Guard Turnover to Assist Ratio (basketball)
   132 turnovers for 410 assists
   369 turnovers for 1120 assists
Solve each proportion.

39. \( \frac{x}{3} = \frac{7}{18} \)
40. \( \frac{x}{-5} = \frac{-6}{17} \)
41. \( \frac{y}{4} = \frac{-8}{15} \)
42. \( \frac{y}{6} = \frac{-7}{20} \)
43. \( \frac{4}{3} = \frac{y}{10} \)
44. \( \frac{5}{-4} = \frac{y}{15} \)
45. \( \frac{-5}{a} = \frac{4}{1.2} \)
46. \( \frac{-6}{a} = \frac{5}{2.4} \)
47. \( \frac{-7}{12} = \frac{z}{8} \)
48. \( \frac{4}{5} = \frac{z}{-6} \)
49. \( \frac{18}{w} = \frac{7}{14} \)
50. \( \frac{-7}{18} = \frac{w}{15} \)
51. \( \frac{4}{6} = \frac{9}{b} \)
52. \( \frac{7}{8} = \frac{21}{b} \)
53. \( \frac{5}{10} = \frac{12}{t} \)
54. \( \frac{8}{17} = \frac{8}{t} \)
55. \( \frac{x+1}{3} = \frac{x-2}{4} \)
56. \( \frac{x-3}{5} = \frac{x-5}{6} \)
57. \( \frac{2x+1}{3} = \frac{3x-2}{5} \)
58. \( \frac{4x-3}{8} = \frac{x+3}{4} \)

Solve the following proportion applications. Be sure to answer the question with the appropriate types of numbers (fraction, decimal, rounded).

59. The property tax rate in a certain state is $1.25 for every $110 in property value. If a home has a property value of $85,000, how much will be the property tax?

60. The property tax rate in a certain state is $1.75 for every $120 in property value. If a home has a property value of $268,000, how much will be the property tax?

61. The slip fees for Ross’ 30 foot sloop sailboat are $195. At this rate, what would the slip fees be for Brad’s 42 foot motor yacht?

62. The slip fees for Tony’s 34 foot yacht are $246.50. At this rate, what would the slip fees be for Sal’s 61 foot ketch sailboat?

63. Carolyn paid $5,760 income tax for $32,000 in income. At this rate, what would her tax be for $45,000 in income?

64. Mark paid $12,880 income tax for $46,000 in income. At this rate, what would his tax be for $65,000 in income?
65. The price for Bobby’s 10 oz hamburger is $5.80. At this rate, what would the price be for Bobby’s 14 oz super hamburger?

66. Rufus’ commission for selling a $12,000 ski boat is $2,160. At this rate, what would his commission be for selling a $38,000 sailboat?

67. A baseball pitcher gives up 13 earned runs for every 54 innings pitched. At this rate, how many earned runs will he give up for 216 innings pitched?

68. How many earned runs will the pitcher from Exercise 67 give up for 9 innings pitched? Round your answer to two decimal places. This is called the pitcher’s earned run average.

69. A basketball point guard has 12 assists for every 5 turnovers. At his rate, how many assists will he have for a season in which he has 145 turnovers?

70. The point guard (from Exercise 65) has 12 assists for every 5 turnovers. At this rate, how many turnovers will he have for a season in which he has 624 assists?

71. A map uses a scale of $\frac{1}{4}$ inch = 30 miles. On a map, two cities are $2\frac{7}{8}$ inches apart. How many miles apart are the two cities?

72. A map uses a scale of $\frac{1}{3}$ inch = 50 miles. On a map, two cities are $4\frac{1}{6}$ inches apart. How many miles apart are the two cities?

73. Using the map scale from Exercise 71, two cities are 420 miles apart. On the map, how many inches apart are the two cities?

74. Using the map scale from Exercise 72, two cities are 650 miles apart. On the map, how many inches apart are the two cities?

75. A car drives 207 miles in 3 hours. At this speed, how far will the car travel in 5.6 hours?

76. A car drives 150 miles in 2.5 hours. At this speed, how far will the car travel in 4.8 hours?

77. Using the car speed from Exercise 75, how long will it take the car to travel 552 miles?

78. Using the car speed from Exercise 76, how long will it take the car to travel 450 miles?

If measured at the same time, the ratio of the height of an object and the length of its shadow is constant. Use this concept to solve the following proportion problems. Assume all measurements are taken at the same time.

79. If a 6 foot fence casts a 4.5 foot shadow, how long would the shadow be for a 30 foot flagpole?

80. If a 15 foot sailboat mast casts a 8 foot shadow, how long would the shadow be for a 25 foot mast?
81. If a 5 foot person casts a 3.2 foot shadow, how tall is a person who casts a 4.8 foot shadow?

82. If a 3 foot child casts a 1.4 foot shadow, how tall is an adult who casts a 2.6 foot shadow?