4.3 Equations of the Form $ax + b = c$

In the first two sections of this chapter we studied expressions. We now turn our attention to equations in algebra. Recall that equations are statements which, when the variable is replaced with a number, can be determined to be true or false. The number which results in a true statement in an equation is called the solution to the equation. We have seen numerous examples of solutions to equations in this book, but how do we obtain solutions to equations? As you can imagine, the key lies in developing the appropriate properties of equations. Consider the following two properties of equations:

**Addition Property of Equations**: If $A = B$, then $A + C = B + C$.

**Multiplication Property of Equations**: If $A = B$ and $C \neq 0$, then $A \cdot C = B \cdot C$.

The first property states that we can add the same quantity to each side of an equation, while the second property states that we can multiply the same nonzero quantity to each side of an equation. Let’s begin with the first property. Suppose we are given the equation:

$$x + 8 = -7$$

To solve the equation, we want to isolate the $x$ variable. To eliminate 8, we will add $-8$ to each side of the equation:

$$x + 8 + (-8) = -7 + (-8)$$

Using the inverse property of addition:

$$x + 0 = -15$$

Now using the identity property of addition:

$$x = -15$$

As a final verification, substitute the solution into the equation:

$$-15 + 8 = -7$$

$$-7 = -7$$

Thus the key to application of the Addition property of equations is actually an earlier property, the Inverse property of addition. A brief summary: we add the opposite of a number (or expression) to each side of an equation if we want to eliminate that quantity from the equation. The following example will help to illustrate the use of this property.
Example 1  Use the Addition property of equations to solve each equation. Include a check of your solution.

a. \( x + 15 = -3 \)

b. \( y + (-4) = -11 \)

c. \( x - 9 = -6 \)

d. \( x - (-6) = 7 \)

Solution  a. We will add \(-15\) (the opposite of 15) to each side of the equation:

\[
\begin{align*}
    x + 15 &= -3 \\
    x + 15 + (-15) &= -3 + (-15) \\
    x + 0 &= -18 \\
    x &= -18
\end{align*}
\]

Checking our solution:

\[
\begin{align*}
    -18 + 15 &= -3 \\
    -3 &= -3
\end{align*}
\]

b. We will add 4 (the opposite of \(-4\)) to each side of the equation:

\[
\begin{align*}
    y + (-4) &= -11 \\
    y + (-4) + 4 &= -11 + 4 \\
    y + 0 &= -7 \\
    y &= -7
\end{align*}
\]

Checking our solution:

\[
\begin{align*}
    -7 + (-4) &= -11 \\
    -11 &= -11
\end{align*}
\]
c. First change the subtraction to addition in the equation:
\[ x - 9 = -6 \]
\[ x + (-9) = -6 \]
We will add 9 (the opposite of \(-9\)) to each side of the equation:
\[ x - 9 = -6 \]
\[ x + (-9) = -6 \]
\[ x + (-9) + 9 = -6 + 9 \]
\[ x + 0 = 3 \]
\[ x = 3 \]
Checking our solution:
\[ 3 - 9 = -6 \]
\[ 3 + (-9) = -6 \]
\[ -6 = -6 \]

d. First change the subtraction to addition in the equation:
\[ x - (-6) = 7 \]
\[ x + 6 = 7 \]
We will add \(-6\) (the opposite of 6) to each side of the equation:
\[ x - (-6) = 7 \]
\[ x + 6 = 7 \]
\[ x + 6 + (-6) = 7 + (-6) \]
\[ x + 0 = 1 \]
\[ x = 1 \]
Checking our solution:
\[ 1 - (-6) = 7 \]
\[ 1 + 6 = 7 \]
\[ 7 = 7 \]
We now turn our attention to the second property of equations. Suppose we are given the equation:

\[ 6x = -9 \]

Note that adding \(-6\) to each side of the equation is not helpful, since \(6x\) and \(-6\) are not like terms, and thus cannot be combined. However, using the Multiplication property of equations, we multiply each side of the equation by \(\frac{1}{6}\):

\[
\frac{1}{6} \cdot 6x = \frac{1}{6} \cdot (-9)
\]

Using the inverse property of multiplication:

\[
1 \cdot x = \frac{-3}{2}
\]

Now using the identity property of multiplication:

\[
x = \frac{-3}{2}
\]

As a final verification, we check our solution in the original equation:

\[
6 \cdot \left( \frac{-3}{2} \right) = -9
\]

\[-9 = -9
\]

Thus the key to application of the Multiplication property of equations is actually an earlier property, the Inverse property of multiplication. A briefer summary: we multiply the reciprocal of a number to each side of an equation if we want to eliminate that quantity from the equation. The following example will help to illustrate the use of this property.
Example 2  Use the Multiplication property of equations to solve each equation. Include a check of your solution.

a. $5x = -30$

b. $-4x = -14$

c. $\frac{2}{3}x = -8$

d. $\frac{x}{6} = -2$

Solution

a. We will multiply by $\frac{1}{5}$ (the reciprocal of 5) to each side of the equation:

$5x = -30$

$\frac{1}{5} \cdot 5x = \frac{1}{5} \cdot (-30)$

$1 \cdot x = -6$

$x = -6$

Checking our solution:

$5(-6) = -30$

$-30 = -30$

b. We will multiply by $-\frac{1}{4}$ (the reciprocal of $-4$) to each side of the equation:

$-4x = -14$

$-\frac{1}{4} \cdot (-4x) = -\frac{1}{4} \cdot (-14)$

$1 \cdot x = \frac{7}{2}$

$x = \frac{7}{2}$

Checking our solution:

$-4 \cdot \left(\frac{7}{2}\right) = -14$

$-14 = -14$
c. We will multiply by $\frac{3}{2}$ (the reciprocal of $\frac{2}{3}$) to each side of the equation:

$$\frac{2}{3}x = -8$$

$$\frac{3}{2} \cdot \frac{2}{3}x = \frac{3}{2} \cdot (-8)$$

$$1 \cdot x = -12$$

$$x = -12$$

Checking our solution:

$$\frac{2}{3} \cdot (-12) = -8$$

$$-8 = -8$$

d. First note that the equation is identical to $\frac{1}{6}x = -2$. We will multiply by 6 (the reciprocal of $\frac{1}{6}$) to each side of the equation:

$$\frac{x}{6} = -2$$

$$\frac{1}{6}x = -2$$

$$6 \cdot \frac{1}{6}x = 6 \cdot (-2)$$

$$1 \cdot x = -12$$

$$x = -12$$

Checking our solution:

$$\frac{-12}{6} = -2$$

$$-2 = -2$$
We now consider equations in which both properties of equations need to be applied in the same problem. Consider the equation:

\[ 2x + 1 = -7 \]

From the first two examples, it seems clear that we need to add \(-1\) (the opposite of 1) and multiply by \(\frac{1}{2}\) (the reciprocal of 2). But in what order to we perform these operations. Note that if we choose to multiply by \(\frac{1}{2}\) first, we would have the equation \(\frac{1}{2}(2x + 1) = \frac{1}{2}(-7)\), which would require the use of the distributive property and working with fractions. Though the equation can be solved that way, a much easier approach is to add \(-1\) first:

\[
\begin{align*}
2x + 1 + (-1) &= 7 + (-1) \\
2x + 0 &= 6 \\
2x &= 6
\end{align*}
\]

Note that we used the inverse and identity properties of addition in the second and third steps. Now we multiply by \(\frac{1}{2}\) to solve the equation:

\[
\begin{align*}
\frac{1}{2} \cdot 2x &= \frac{1}{2} \cdot 6 \\
1 \cdot x &= 3 \\
x &= 3
\end{align*}
\]

Again note the use of the inverse and identity properties of multiplication in the second and third steps. Finally, we check our answer:

\[
\begin{align*}
2 \cdot (3) + 1 &= 7 \\
6 + 1 &= 7 \\
7 &= 7
\end{align*}
\]

As a summary, when solving equations of the form \(ax + b = c\), we first add \(-b\) to each side of the equation (the opposite of \(b\)), and then multiply the resulting equation by \(\frac{1}{a}\) (the reciprocal of \(a\)). The following example should help to illustrate this process.
Example 3  Solve each equation. Include a check of your solution.

a.  \(-3x + 1 = -11\)

b.  \(-4x - 3 = 7\)

c.  \(\frac{1}{2}x + 4 = -3\)

d.  \(\frac{x}{-3} + 1 = -3\)

Solution  

a.  First add \(-1\) (the opposite of 1), then multiply by \(-\frac{1}{3}\) (the reciprocal of \(-3\)), to each side of the equation. The steps are:

\[-3x + 1 = -11\]
\[-3x + 1 + (-1) = -11 + (-1)\]
\[-3x = -12\]
\[-\frac{1}{3} \cdot (-3x) = -\frac{1}{3} \cdot (-12)\]
\[x = 4\]

Note that we left off the inverse property and identity property steps, primarily to save time and space. Checking the solution:

\[-3(4) + 1 = -11\]
\[-12 + 1 = -11\]
\[-11 = -11\]

b.  First add \(3\) (the opposite of \(-3\)), then multiply by \(-\frac{1}{4}\) (the reciprocal of \(-4\)), to each side of the equation. The steps are:

\[-4x - 3 = 7\]
\[-4x - 3 + 3 = 7 + 3\]
\[-4x = 10\]
\[-\frac{1}{4} \cdot (-4x) = -\frac{1}{4} \cdot (10)\]
\[x = -\frac{5}{2}\]
Now checking the solution:
\[-4 \cdot \left( -\frac{5}{2} \right) - 3 = 7\]
\[10 - 3 = 7\]
\[7 = 7\]

c. First add \(-4\) (the opposite of 4), then multiply by 2 (the reciprocal of \(\frac{1}{2}\)), to each side of the equation. The steps are:
\[
\frac{1}{2}x + 4 = -3
\]
\[
\frac{1}{2}x + 4 + (-4) = -3 + (-4)
\]
\[
\frac{1}{2}x = -7
\]
\[
2 \cdot \frac{1}{2}x = 2 \cdot (-7)
\]
\[x = -14\]

Now checking the solution:
\[
\frac{1}{2}(-14) + 4 = -3
\]
\[-7 + 4 = -3\]
\[-3 = -3\]
d. First note that the equation is equivalent to \( \frac{-1}{3}x + 1 = -3 \). First add \(-1\) (the opposite of 1), then multiply by \(-3\) (the reciprocal of \(\frac{-1}{3}\)), to each side of the equation. The steps are:

\[
-\frac{1}{3}x + 1 = -3
\]

\[
-\frac{1}{3}x + 1 + (-1) = -3 + (-1)
\]

\[
-\frac{1}{3}x = -4
\]

\[
-3 \cdot \left(-\frac{1}{3}x\right) = -3 \cdot (-4)
\]

\[
x = 12
\]

Now checking the solution:

\[
-\frac{1}{3}(12) + 1 = -3
\]

\[
-4 + 1 = -3
\]

\[
-3 = -3
\]

In the previous example, we solved the equation:

\[
\frac{1}{2}x + 4 = -3
\]

An alternate approach is to first multiply by 2, then solve the resulting equation:

\[
\frac{1}{2}x + 4 = -3
\]

\[
2\left(\frac{1}{2}x + 4\right) = 2(-3)
\]

\[
2 \cdot \frac{1}{2}x + 2 \cdot 4 = -6
\]

\[
x + 8 = -6
\]

\[
x + 8 + (-8) = -6 + (-8)
\]

\[
x = -14
\]
Note that the solution is exactly the same. This alternate approach has the advantage of eliminating fractions from an equation as the first step. Consider the more complicated equation:

\[
\frac{1}{3}x + \frac{1}{2} = -1
\]

Is there a number we could multiply by to eliminate fractions? Though 3 would work to eliminate the fraction \(\frac{1}{3}\), it would not eliminate the second fraction. Since the LCM of the two denominators is 6, we will multiply by 6 in the equation:

\[
6\left(\frac{1}{3}x + \frac{1}{2}\right) = 6(-1)
\]

\[
6\cdot\frac{1}{3}x + 6\cdot\frac{1}{2} = -6
\]

\[
2x + 3 = -6
\]

\[
2x + 3 + (-3) = -6 + (-3)
\]

\[
2x = -9
\]

\[
\frac{1}{2} \cdot 2x = \frac{1}{2} \cdot (-9)
\]

\[
x = \frac{-9}{2}
\]

Checking our solution:

\[
\frac{1}{3} \cdot \left(-\frac{9}{2}\right) + \frac{1}{2} = -1
\]

\[
-\frac{3}{2} + \frac{1}{2} = -1
\]

\[-1 = -1
\]

The advantage of this approach is the elimination of fractions from the equation. With practice, you will find this approach to be a bit easier. The next example provides additional demonstrations of this approach.
Example 4  Solve each equation by first eliminating fractions. Include a check of your solution.

a. \( \frac{3}{4}x - 2 = -5 \)

b. \( -\frac{1}{3}x - 1 = \frac{1}{6} \)

c. \( -\frac{1}{2}x + \frac{1}{4} = -\frac{1}{8} \)

d. \( \frac{1}{3}x - \frac{1}{4} = -\frac{1}{6} \)

Solution  

a. Multiply each side of the equation by 4 to clear the equation of fractions:

\[
4 \cdot \left( \frac{3}{4}x - 2 \right) = 4 \cdot (-5)
\]

\[
4 \cdot \frac{3}{4}x - 4 \cdot 2 = -20
\]

\[
3x - 8 = -20
\]

\[
3x - 8 + 8 = -20 + 8
\]

\[
3x = -12
\]

\[
\frac{1}{3} \cdot 3x = \frac{1}{3} \cdot (-12)
\]

\[
x = -4
\]

Checking the solution:

\[
\frac{3}{4}(-4) - 2 = -5
\]

\[-3 - 2 = -5
\]

\[-5 = -5
\]
b. The LCM of 3 and 6 is 6. Multiply each side of the equation by 6 to clear the equation of fractions:

\[-\frac{1}{3}x - 1 = \frac{1}{6}\]

\[6\left(-\frac{1}{3}x - 1\right) = 6 \cdot \frac{1}{6}\]

\[6\left(-\frac{1}{3}x\right) - 6 \cdot 1 = 1\]

\[-2x - 6 = 1\]

\[-2x - 6 + 6 = 1 + 6\]

\[-2x = 7\]

\[-\frac{1}{2} \cdot (-2x) = -\frac{1}{2} \cdot (7)\]

\[x = -\frac{7}{2}\]

Checking the solution:

\[-\frac{1}{3} \cdot \left(-\frac{7}{2}\right) - 1 = \frac{1}{6}\]

\[\frac{7}{6} - 1 = \frac{1}{6}\]

\[\frac{7}{6} - \frac{6}{6} = \frac{1}{6}\]

\[\frac{1}{6} = \frac{1}{6}\]
c. The LCM of 2, 4, and 8 is 8. Multiply each side of the equation by 8 to clear the equation of fractions:

\[ \frac{-1}{2}x + \frac{1}{4} = -\frac{1}{8} \]

\[ 8 \cdot \left( \frac{-1}{2}x + \frac{1}{4} \right) = 8 \cdot \left( -\frac{1}{8} \right) \]

\[ 8 \cdot \left( -\frac{1}{2}x \right) + 8 \cdot \left( \frac{1}{4} \right) = 8 \cdot \left( -\frac{1}{8} \right) \]

\[ -4x + 2 = -1 \]

\[ -4x + 2 + (-2) = -1 + (-2) \]

\[ -4x = -3 \]

\[ -\frac{1}{4} \cdot (-4x) = -\frac{1}{4} \cdot (-3) \]

\[ x = \frac{3}{4} \]

Checking the solution:

\[ -\frac{1}{2} \cdot \left( \frac{3}{4} \right) + \frac{1}{4} = -\frac{1}{8} \]

\[ -\frac{3}{8} + \frac{1}{4} = -\frac{1}{8} \]

\[ -\frac{3}{8} + \frac{2}{8} = -\frac{1}{8} \]

\[ -\frac{1}{8} = -\frac{1}{8} \]
d. The LCM of 3, 4, and 6 is 12. Multiply each side of the equation by 12 to clear the equation of fractions:

$$\frac{1}{3}x - \frac{1}{4} = -\frac{1}{6}$$

$$12 \cdot \left( \frac{1}{3}x - \frac{1}{4} \right) = 12 \cdot \left( -\frac{1}{6} \right)$$

$$12 \cdot \frac{1}{3}x - 12 \cdot \frac{1}{4} = -2$$

$$4x - 3 = -2$$

$$4x - 3 + 3 = -2 + 3$$

$$4x = 1$$

$$\frac{1}{4} \cdot (4x) = \frac{1}{4} \cdot (1)$$

$$x = \frac{1}{4}$$

Checking the solution:

$$\frac{1}{3} \cdot \left( \frac{1}{4} \right) - \frac{1}{4} = -\frac{1}{6}$$

$$\frac{1}{12} - \frac{1}{4} = -\frac{1}{6}$$

$$\frac{1}{12} - \frac{3}{12} = -\frac{1}{6}$$

$$\frac{1}{6} = -\frac{1}{6}$$

In the next section we will continue the study of equations with equations with variables on each side of the equality symbol.

Terminology

- equation
- solution
- Addition property of equations
- Multiplication property of equations
Exercise Set 4.3

Use the Addition property of equations to solve each equation. Include a check of your solution.

1. \(x + 12 = 4\)
2. \(x + 8 = 2\)
3. \(x - 5 = -2\)
4. \(x - 9 = -4\)
5. \(y + 6 = -3\)
6. \(y + 8 = -5\)
7. \(a - 9 = -16\)
8. \(a - 8 = -13\)
9. \(v + (-3) = -7\)
10. \(v + (-8) = -13\)
11. \(b + (-7) = -3\)
12. \(b + (-11) = -6\)
13. \(x - (-5) = -2\)
14. \(x - (-7) = -3\)
15. \(x - (-9) = 4\)
16. \(x - (-12) = 7\)
17. \(x + \frac{2}{3} = \frac{1}{2}\)
18. \(x + \frac{3}{4} = \frac{1}{3}\)
19. \(x - \frac{1}{3} = -\frac{5}{6}\)
20. \(x - \frac{1}{2} = -\frac{3}{4}\)
21. \(y + 3.2 = 1.17\)
22. \(y + 6.7 = 2.38\)
23. \(t - 4.57 = -8\)
24. \(t - 6.82 = -9.1\)

Use the Multiplication property of equations to solve each equation. Include a check of your solution.

25. \(6x = -42\)
26. \(4x = -64\)
27. \(5y = -75\)
28. \(9y = 108\)
29. \(-7s = -56\)
30. \(-9s = -99\)
31. \(8t = -20\)
32. \(6t = -33\)
33. \(-4b = 18\)
34. \(-9b = 42\)
35. \(-6a = -27\)
36. \(-12a = -4\)
37. \(\frac{2}{3}x = -12\)
38. \(-\frac{2}{3}x = -12\)
39. \(-\frac{3}{4}x = -9\)
40. \(\frac{3}{4}x = -9\)
41. \(\frac{5}{6}y = \frac{3}{4}\)
42. \(\frac{2}{3}y = \frac{10}{9}\)
43. \(\frac{4}{5}a = -\frac{8}{15}\)
44. \(-\frac{2}{5}a = -\frac{8}{15}\)
45. \(\frac{x}{5} = -3\)
46. \(\frac{x}{4} = -7\)
Solve each equation. Include a check of your solution.

53. \(2x + 1 = 13\)
55. \(3y - 4 = 11\)
57. \(4a + 3 = -9\)
59. \(5b - 2 = -27\)
61. \(-2x + 3 = -7\)
63. \(-4t - 1 = -17\)
65. \(-3u + 1 = -6\)
67. \(2v + 3 = -4\)
69. \(-4x + 3 = 1\)
71. \(2y - 3 = -6\)
73. \(\frac{1}{2}x - 2 = 1\)
74. \(\frac{1}{2}x + 2 = -3\)
75. \(-\frac{1}{3}y + 3 = -1\)
76. \(-\frac{1}{3}y - 2 = -6\)
77. \(-\frac{1}{4}a + 3 = 1\)
78. \(-\frac{1}{5}a + 4 = -2\)
79. \(\frac{x}{2} + 3 = -1\)
80. \(\frac{x}{3} + 5 = -2\)
81. \(\frac{y}{4} - 3 = -7\)
82. \(\frac{y}{5} - 5 = 1\)
83. \(\frac{a}{2} - 4 = -8\)
84. \(\frac{a}{-3} - 5 = -9\)
85. \(\frac{b}{-4} + 3 = 8\)
86. \(\frac{b}{-5} + 1 = 7\)
87. \(0.3x + 4.2 = -1.8\)
88. \(0.2x + 1.4 = -0.8\)
89. \(-0.5x + 3.2 = -2.7\)
90. \(-0.6x + 4.2 = 0.6\)
Solve each equation by first eliminating fractions. Include a check of your solution.

91. \( \frac{2}{3}x - 1 = -4 \)
92. \( -\frac{2}{3}x - 5 = -7 \)
93. \( -\frac{1}{3}y + 2 = \frac{2}{3} \)
94. \( -\frac{1}{4}y + 1 = \frac{1}{2} \)
95. \( -\frac{1}{3}a + \frac{1}{2} = -\frac{1}{6} \)
96. \( -\frac{1}{4}a + \frac{1}{2} = -\frac{3}{8} \)
97. \( -\frac{1}{3}b + \frac{1}{4} = -\frac{5}{6} \)
98. \( -\frac{1}{4}b + \frac{3}{2} = -\frac{2}{3} \)
99. \( -\frac{3}{4}s + \frac{1}{2} = -\frac{1}{3} \)
100. \( -\frac{5}{8}s + \frac{3}{4} = -\frac{1}{2} \)
101. \( -\frac{7}{12}t + \frac{2}{3} = -\frac{3}{4} \)
102. \( -\frac{5}{12}t + \frac{1}{3} = -\frac{1}{6} \)