3.4 Multiplication and Division of Rational Numbers

We now turn our attention to multiplication and division with both fractions and decimals. Consider the multiplication problem:

\[
\frac{5}{8} \cdot \frac{12}{25}
\]

One approach is to multiply numerators and multiply denominators, resulting in the fraction \(\frac{60}{200}\), which can then be reduced by either GCF or by using primes. However, a faster approach is to consider the reducing (using primes or GCF) as part of the multiplication process. Here are the steps using the GCF method:

\[
\frac{5}{8} \cdot \frac{12}{25} = \frac{5 \cdot 12}{8 \cdot 25} = \frac{5 \cdot 4 \cdot 3}{4 \cdot 2 \cdot 5 \cdot 5}
\]

factoring the GCF

\[
= \frac{5 \cdot 4 \cdot 3}{4 \cdot 2 \cdot 5 \cdot 5}
\]

combining factors in one fraction

\[
= \frac{3}{2 \cdot 5}
\]

write remaining factors after cancelling

\[
= \frac{3}{10}
\]

multiply remaining factors

Using the prime factors method makes it a bit easier to organize the factoring:

\[
\frac{5}{8} \cdot \frac{12}{25} = \frac{5 \cdot 12}{8 \cdot 25} = \frac{5 \cdot 2 \cdot 2 \cdot 3}{2 \cdot 2 \cdot 5 \cdot 5}
\]

factoring into primes

\[
= \frac{2 \cdot 2 \cdot 3 \cdot 5}{2 \cdot 2 \cdot 5 \cdot 5}
\]

combining factors in one fraction

\[
= \frac{3}{2 \cdot 5}
\]

write remaining factors after cancelling

\[
= \frac{3}{10}
\]

multiply remaining factors
We will use this second method (using prime numbers) throughout this section, since it is easier to organize problems with prime numbers. Both methods are equivalent, and produce identical answers. Note that your final answer should not need to be simplified, since simplifying the fractions is actually part of the process involved with the multiplication.

**Example 1** Multiply the given rational numbers.

- **a.** \( \frac{7}{12} \cdot \frac{9}{14} \)
- **b.** \( \frac{-9}{20} \cdot \frac{25}{27} \)
- **c.** \( \frac{-5}{6} \cdot \frac{-8}{9} \left( \frac{-12}{25} \right) \)
- **d.** \( \frac{5}{8x} \cdot \frac{16x}{15y} \)

**Solution**

- **a.** Following the steps outlined in the prime method above:
  \[
  \frac{7 \cdot 9}{12 \cdot 14} = \frac{7 \cdot 3 \cdot 3}{2 \cdot 2 \cdot 3 \cdot 2 \cdot 7} = \frac{3 \cdot 3 \cdot 7}{2 \cdot 2 \cdot 2 \cdot 3 \cdot 7} = \frac{3}{2 \cdot 2 \cdot 2} = \frac{3}{8}
  \]
  multiply remaining factors

- **b.** Note that the answer will be negative, since one negative is involved in the multiplication. Following the same steps:
  \[
  \frac{-9 \cdot 25}{20 \cdot 27} = \frac{-3 \cdot 3 \cdot 5 \cdot 5}{2 \cdot 2 \cdot 5 \cdot 3 \cdot 3 \cdot 3} = \frac{-5 \cdot 5}{2 \cdot 2 \cdot 3 \cdot 3 \cdot 3} = \frac{-5}{2 \cdot 2 \cdot 3} = \frac{-5}{12}
  \]
  multiply remaining factors
c. Note that the answer will be positive, since two negatives are involved in the multiplication. Following the same steps:

\[ \frac{5}{6} \cdot \frac{8}{9} \cdot \left( -\frac{12}{25} \right) \]

\[ = \frac{5 \cdot 8 \cdot 12}{6 \cdot 9 \cdot 25} \]

writing the product as a positive

\[ = \frac{5 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 3 \cdot 3 \cdot 5 \cdot 5}{2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 3 \cdot 5 \cdot 5} \]

factoring into primes

\[ = \frac{2 \cdot 2 \cdot 2 \cdot 2 \cdot 3 \cdot 3 \cdot 5 \cdot 5}{2 \cdot 2 \cdot 2 \cdot 2 \cdot 3 \cdot 5 \cdot 5} \]

combining factors in one fraction

\[ = \frac{2 \cdot 2 \cdot 2 \cdot 2}{3 \cdot 3 \cdot 5} \]

write remaining factors after cancelling

\[ = \frac{16}{45} \]

multiply remaining factors

d. Don’t get confused with the variables. Following the same steps:

\[ \frac{5}{8x} \cdot \frac{16x}{15y} \]

\[ = \frac{5 \cdot 2 \cdot 2 \cdot 2 \cdot x}{2 \cdot 2 \cdot 2 \cdot 2 \cdot 3 \cdot 5 \cdot x \cdot y} \]

factoring into primes

\[ = \frac{2 \cdot 2 \cdot 2 \cdot 2 \cdot 5 \cdot x}{2 \cdot 2 \cdot 2 \cdot 3 \cdot 5 \cdot x \cdot y} \]

combining factors in one fraction

\[ = \frac{2}{3 \cdot y} \]

write remaining factors after cancelling

\[ = \frac{2}{3y} \]

multiply remaining factors

To understand division of fractions, we need to go back to whole numbers. We know that

\[ 6 \div 2 = 3 \], and also that \[ 6 \cdot \frac{1}{2} = \frac{6}{2} = 3 \], so \[ 6 \div 2 = 6 \cdot \frac{1}{2} \]. Also \[ 12 \div 3 = 4 \] and \[ 12 \cdot \frac{1}{3} = \frac{12}{3} = 4 \], so \[ 12 \div 3 = 12 \cdot \frac{1}{3} \]. Thus dividing numbers is equivalent to multiplying by another fraction, called the **reciprocal** of the second number. In general:

\[ x \div y = x \cdot \frac{1}{y} \]
Here \( \frac{1}{y} \) is called the reciprocal of \( y \). Applying this same principle to fractions, we have a rule for division:

\[
\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \cdot \frac{d}{c} = \frac{ad}{bc}
\]

Here \( \frac{d}{c} \) is called the reciprocal of \( \frac{c}{d} \). In order to divide \( -\frac{3}{4} \) by \( \frac{5}{8} \), we therefore convert to the multiplication \( -\frac{3}{4} \cdot \frac{8}{5} \). Completing the steps:

\[
-\frac{3}{4} \div \frac{5}{8} = -\frac{3}{4} \cdot \frac{8}{5}
\]

converting to division

\[
= -\frac{3 \cdot 2 \cdot 2}{2 \cdot 2 \cdot 5}
\]

factoring into primes

\[
= -\frac{2 \cdot 2 \cdot 2 \cdot 3}{2 \cdot 2 \cdot 5}
\]

combining factors in one fraction

\[
= -\frac{2 \cdot 3}{5}
\]

write remaining factors after cancelling

\[
= -\frac{6}{5}
\]

multiply remaining factors

After the first step of division, the problem is identical to multiplication.

**Example 2** Divide the given rational numbers.

a. \( \frac{9}{16} + \left( -\frac{3}{4} \right) \)

b. \( -\frac{25}{36} + \left( -\frac{5}{9} \right) \)

c. \( -\frac{5}{6} + (-10) \)

d. \( \frac{8xy}{9a^2b} + \frac{16xy^2}{27ab^3} \)
**Solution**

a. Rewrite the division as a multiplication, then use prime factors to simplify the product. The steps are:

\[
\frac{9}{16} \div \left( -\frac{3}{4} \right) = \frac{9}{16} \cdot \left( -\frac{4}{3} \right) \quad \text{converting to division}
\]

\[
= -\frac{3 \cdot 3 \cdot 2 \cdot 2}{2 \cdot 2 \cdot 2 \cdot 3} \quad \text{factoring into primes}
\]

\[
= -\frac{2 \cdot 2 \cdot 3 \cdot 3}{2 \cdot 2 \cdot 2 \cdot 3} \quad \text{combining factors in one fraction}
\]

\[
= -\frac{3}{2 \cdot 2} \quad \text{write remaining factors after cancelling}
\]

\[
= -\frac{3}{4} \quad \text{multiply remaining factors}
\]

b. Rewrite the division as a multiplication, then use prime factors to simplify the product. The steps are:

\[
-\frac{25}{36} \div \left( -\frac{5}{9} \right) = -\frac{25}{36} \cdot \left( -\frac{9}{5} \right) \quad \text{converting to division}
\]

\[
= \frac{5 \cdot 5 \cdot 3 \cdot 3}{2 \cdot 2 \cdot 3 \cdot 3} \quad \text{factoring into primes}
\]

\[
= \frac{3 \cdot 3 \cdot 5 \cdot 5}{2 \cdot 2 \cdot 3 \cdot 3 \cdot 5} \quad \text{combining factors in one fraction}
\]

\[
= \frac{5}{2 \cdot 2} \quad \text{write remaining factors after cancelling}
\]

\[
= \frac{5}{4} \quad \text{multiply remaining factors}
\]
c. Note that the divisor is a fraction, namely \( -\frac{10}{1} \). Rewrite the division as a multiplication, then use prime factors to simplify the product. The steps are:

\[
\frac{5}{6} \div \left( -\frac{10}{1} \right) = \frac{5}{6} \cdot \left( -\frac{1}{10} \right) \quad \text{converting to division}
\]

\[
= \frac{5}{6} \cdot \frac{1}{2 \cdot 3 \cdot 2 \cdot 5} \quad \text{factoring into primes}
\]

\[
= \frac{1 \cdot 5}{2 \cdot 2 \cdot 3 \cdot 5} \quad \text{combining factors in one fraction}
\]

\[
= \frac{1}{2 \cdot 2 \cdot 3} \quad \text{write remaining factors after cancelling}
\]

\[
= \frac{1}{12} \quad \text{multiply remaining factors}
\]

d. Just treat variables as prime numbers. Rewrite the division as a multiplication, then use prime factors to simplify the product. The steps are:

\[
\frac{8xy}{9a^2b} \div \frac{16xy^3}{27ab^3} = \frac{8xy}{9a^2b} \cdot \frac{27ab^3}{16xy^3} \quad \text{converting to division}
\]

\[
= \frac{8xy}{9a^2b} \cdot \frac{27ab^3}{16xy^3} \quad \text{factoring into primes}
\]

\[
= \frac{2 \cdot 2 \cdot 2 \cdot x \cdot y \cdot 3 \cdot 3 \cdot 3 \cdot a \cdot b \cdot b \cdot b}{3 \cdot 3 \cdot a \cdot a \cdot b} \quad \text{combining factors in one fraction}
\]

\[
= \frac{2 \cdot 2 \cdot 2 \cdot 3 \cdot 3 \cdot x \cdot y \cdot a \cdot b \cdot b \cdot b}{2 \cdot 2 \cdot 2 \cdot 3 \cdot 3 \cdot x \cdot y \cdot a \cdot a \cdot b}
\]

\[
= \frac{3 \cdot b \cdot b}{2 \cdot y \cdot a} \quad \text{write remaining factors after cancelling}
\]

\[
= \frac{3b^2}{2ay} \quad \text{multiply remaining factors}
\]
When working with mixed numbers (both positive and negative), we can multiply and divide by simply converting them to fractions. For example, to multiply $1 \frac{1}{2} \cdot 2 \frac{1}{3}$, we convert each mixed number to a fraction, then multiply as in the previous examples:

$$1 \frac{1}{2} \cdot 2 \frac{1}{3} = \frac{3 \cdot 7}{2 \cdot 3}$$

converting to fractions

$$= \frac{3 \cdot 7}{2 \cdot 3}$$

factoring into primes

$$= \frac{3 \cdot 7}{2 \cdot 3}$$

cancelling common factors

$$= \frac{7}{2}$$

write remaining factors after cancelling

$$= 3 \frac{1}{2}$$

converting back to mixed form

Division of mixed numbers is similar, except that the division must be rewritten as a multiplication problem.

**Example 3** Perform the following multiplications and divisions. Write all answers as mixed numbers.

a. $-5 \frac{1}{3} \cdot 2 \frac{1}{4}$

b. $-6 \cdot \left(-4 \frac{1}{3}\right)$

c. $1 \frac{1}{8} + \left(-3 \frac{3}{4}\right)$

d. $-6 \frac{3}{4} + \left(-3 \frac{3}{5}\right)$
Solution

a. Convert the mixed numbers to fractions, perform the multiplication, then convert back to mixed numbers. The steps are:

\[-5 \frac{1}{3} \cdot 2 \frac{1}{4} = \frac{-16 \cdot 9}{3 \cdot 4}\]

converting to fractions

\[= -\frac{2 \cdot 2 \cdot 2 \cdot 2 \cdot 3 \cdot 3}{2 \cdot 2} \frac{3}{3}\]

factoring into primes

\[= -\frac{2 \cdot 2 \cdot 2 \cdot 2 \cdot 3}{2 \cdot 2 \cdot 3}\]

cancelling common factors

\[= -\frac{2 \cdot 2 \cdot 3}{1}\]

write remaining factors after cancelling

\[= -12\]

converting back to mixed form

b. Convert the mixed numbers to fractions, perform the multiplication, then convert back to mixed numbers. The steps are:

\[-6 \cdot \left(-4 \frac{1}{3}\right) = \frac{6 \cdot 13}{1 \cdot 3}\]

converting to fractions

\[= \frac{2 \cdot 3 \cdot 13}{1 \cdot 3}\]

factoring into primes

\[= \frac{2 \cdot 3 \cdot 13}{1 \cdot 3}\]

cancelling common factors

\[= \frac{2 \cdot 13}{1}\]

write remaining factors after cancelling

\[= 26\]

converting back to mixed form
c. Convert the mixed numbers to fractions, convert the division to multiplication, perform the multiplication, then convert back to mixed numbers. The steps are:

\[
1 \frac{1}{8} \div \left( -3 \frac{3}{4} \right) = \frac{9}{8} \div \left( -\frac{15}{4} \right)
\]

converting to fractions

\[
= \frac{9}{8} \cdot \left( -\frac{4}{15} \right)
\]

converting to multiplication

\[
= -\frac{3 \cdot 3 \cdot 2 \cdot 2}{2 \cdot 2 \cdot 2 \cdot 3 \cdot 5}
\]

factoring into primes

\[
= -\frac{2 \cdot 2 \cdot 3 \cdot 3}{2 \cdot 2 \cdot 2 \cdot 3 \cdot 5}
\]

cancelling common factors

\[
= -\frac{3}{2 \cdot 5}
\]

write remaining factors after cancelling

\[
= -\frac{3}{10}
\]

multiplying factors

d. Convert the mixed numbers to fractions, convert the division to multiplication, perform the multiplication, then convert back to mixed numbers. The steps are:

\[
-6 \frac{1}{4} \div \left( -3 \frac{3}{5} \right) = -\frac{25}{4} \div \left( -\frac{18}{5} \right)
\]

converting to fractions

\[
= -\frac{25}{4} \cdot \left( -\frac{5}{18} \right)
\]

converting to multiplication

\[
= \frac{5 \cdot 5 \cdot 5}{2 \cdot 2 \cdot 2 \cdot 3 \cdot 3}
\]

factoring into primes

\[
= \frac{5 \cdot 5 \cdot 5}{2 \cdot 2 \cdot 2 \cdot 3 \cdot 3}
\]

combining factors

\[
= \frac{125}{72}
\]

multiplying factors

\[
= \frac{153}{72}
\]

converting to mixed number
We now turn our attention to multiplication and division with decimals. You probably remember that we need to count digits after the decimal point to perform multiplication of decimals. Suppose we are multiplying the decimals:

$$5.2 \times 0.34$$

If we first convert each decimal to a fraction, we have:

$$5.2 \times 0.34 = \frac{52}{10} \times \frac{34}{100}$$

Now multiplying numerators and denominators (without simplifying):

$$\frac{52}{10} \times \frac{34}{100} = \frac{52 \times 34}{10 \times 100} = \frac{1768}{1000}$$

Converting this last fraction to a decimal:

$$\frac{1768}{1000} = 1.768$$

The “shortcut” you have learned is readily apparent: Multiply the digits \(52 \times 34\), then count the total number of digits after the decimal point to determine the place value \((2 + 1 = 3)\), and place the decimal point. Multiplying decimals is very much like multiplying whole numbers, with the additional step of placing the decimal point.

**Example 4** Multiply the following decimals.

a. \(0.8 \times 5.7\)

b. \(-3.05 \times 0.62\)

c. \(-5.7 \times (-4.01)\)

d. \(0.000034 \times 2650\)

e. \(0.00053 \times 1000\)
Solution  

a. First multiply the digits: $8 \times 57 = 456$

There are two digits after the decimal place, so our answer should have two digits also. Thus the answer is 4.56.

b. First multiply the digits: $-305 \times 62 = -18910$

There are four digits after the decimal place, so our answer should have four digits also. Thus the answer is $-1.8910$, or $-1.891$ after dropping the place-value holder.

c. First multiply the digits: $-57 \times (-401) = 22857$

There are three digits after the decimal place, so our answer should have three digits also. Thus the answer is 22.857.

d. First multiply the digits: $34 \times 2650 = 90100$

There are six digits after the decimal place, so our answer should have six digits also. Thus the answer is $0.090100$, or $0.0901$ after dropping the last two place-value holders. Note how we had to include a place-value holder before the 9 digit, in order to have six digits after the decimal place.

e. Note that the digits are 53000. There are five digits after the decimal place, so the value is 0.53000, or just 0.53. Note that, when multiplying by a power of 10 (such as 1000), we can quickly do the multiplication by just “moving” the decimal point to the right, once for each 10 being multiplied. Since multiplying by 1000 adds on three zero digits (which count as digits after the decimal place), the effect is to remove three digits after the decimal.

Division of decimals involves a similar technique involving moving the decimal point with the dividing number, called the divisor. Consider the division problem:

$$5.04 \div 0.3$$

Note that the divisor, which is 0.3, contains a decimal. The quotient will be the same if each number is multiplied by the same number. This can be seen by converting the division problem to a fraction problem:

$$5.04 \div 0.3 = \frac{5.04}{0.3}$$
Now multiplying the fraction by the form of 1 which will remove the decimal in the denominator, which in this case is $\frac{10}{10}$:

$$\frac{5.04 \times 10}{0.3 \times 10} = \frac{50.4}{3} \text{ or } 50.4 \div 3$$

Thus the two divisions $5.04 \div 0.3 = 50.4 \div 3$. Now compute the quotient just as if whole numbers were involved (which they are in the divisor):

$$\begin{array}{c|c}
& 16.8 \\
\hline 3 & 50.4 \\
\end{array}$$

$$\begin{array}{c|c}
& 3 \\
\hline & 20 \\
\end{array}$$

$$\begin{array}{c|c}
& 18 \\
\hline & 24 \\
\end{array}$$

$$\begin{array}{c|c}
& 24 \\
\hline & 0 \\
\end{array}$$

Thus, when dividing decimals, if the divisor involves a decimal, multiply by the appropriate power of 10 to eliminate its decimal. Being sure to keep the decimal points lined up, just perform the division as with whole numbers.

**Example 5**  Divide the following decimals.

a. $2.52 \div 12$

b. $3.006 \div 0.3$

c. $0.126 \div 0.06$

d. $3.006 \div 0.24$

e. $51.46 \div 1000$
Solution

a. Since the divisor does not contain a decimal, we can compute the division directly:

\[
\begin{array}{c}
\text{2.52} \\
12 \overline{)2.52} \\
24 \\
12 \\
12 \\
0
\end{array}
\]

The quotient is 0.21.

b. Since the divisor is 0.3, we multiply each number by 10 to change the division problem to \(30.06 \div 3\). Now finding the quotient:

\[
\begin{array}{c}
\text{30.06} \\
3 \overline{)30.06} \\
3 \\
0 \\
0 \\
0 \\
0 \\
6 \\
6 \\
0
\end{array}
\]

The quotient is 10.02.

c. Since the divisor is 0.06, we multiply each number by 100 to change the division problem to \(12.6 \div 6\). Now finding the quotient:

\[
\begin{array}{c}
\text{12.6} \\
6 \overline{)12.6} \\
12 \\
6 \\
6 \\
0
\end{array}
\]

The quotient is 2.1.
d. Since the divisor is 0.24, we multiply each number by 100 to change the division problem to \( \frac{300.6}{24} \). Now finding the quotient:

\[
\begin{array}{c}
24)300.600 \\
24 \\
60 \\
48 \\
126 \\
120 \\
60 \\
48 \\
120 \\
120 \\
0
\end{array}
\]

The quotient is 12.525. Note that we had to add two zeros in the division before the division was complete.

e. Recall in the previous example what occurred when the decimal was multiplied by 1000. By the same reasoning, dividing by 1000 should decrease the place values, or “move” the decimal point to the left three times. Thus the quotient is 0.05146.

Multiplication and division of decimals by powers of ten is very common. The following table summarizes what happens.

<table>
<thead>
<tr>
<th>type</th>
<th>what to do</th>
<th>example</th>
</tr>
</thead>
<tbody>
<tr>
<td>multiplication by (10^n)</td>
<td>move decimal to the right (n) times</td>
<td>(4.381 \times 100 = 438.1)</td>
</tr>
<tr>
<td>division by (10^n)</td>
<td>move decimal to the left (n) times</td>
<td>(62.84 \div 100 = 0.6284)</td>
</tr>
</tbody>
</table>
We finish this section with a topic from Chapter 1 involving division of whole numbers. When presented with the division problem \(57 \div 5\), we have three different types of division to perform. Using a whole number approach:

\[
\begin{array}{c}
\phantom{5}11 \\
5 \overline{)57}
\end{array}
\]

\[
\begin{array}{c}
5 \\
7
\end{array}
\]

\[
\begin{array}{c}
5 \\
2
\end{array}
\]

So \(57 \div 5 = 11\) remainder 2. With mixed numbers, we know this remainder is a portion of 5, so we can write this answer as \(11\frac{2}{5}\). The third approach is to continue the division, adding a 0 place-holder:

\[
\begin{array}{c}
\phantom{5}11.4 \\
5 \overline{)57.0}
\end{array}
\]

\[
\begin{array}{c}
5 \\
7
\end{array}
\]

\[
\begin{array}{c}
5 \\
20
\end{array}
\]

\[
\begin{array}{c}
20 \\
0
\end{array}
\]

So \(57 \div 5 = 11.4\). All of these answers are correct. Which one to use is dependent upon the type of problem (or application), as well as the instructions in the problem. Note that the continued division with decimals often will produce a repeating decimal, which is why we usually prefer the mixed number form for the answer. However, in applications which require a decimal (money, for example), the third approach will still be used.

**Terminology**

- reciprocal
- divisor
Exercise Set 3.4

Multiply the given rational numbers. Leave your answers as fractions.

1. \( \frac{8}{21} \cdot \frac{7}{16} \)
2. \( \frac{9}{10} \cdot \frac{14}{15} \)
3. \( \frac{35}{14} \cdot \frac{14}{25} \)
4. \( \frac{21}{20} \cdot \frac{20}{27} \)
5. \( -\frac{12}{25} \cdot \frac{35}{36} \)
6. \( -\frac{8}{15} \cdot \frac{21}{32} \)
7. \( \frac{13}{14} \cdot \left( -\frac{21}{143} \right) \)
8. \( \frac{25}{32} \cdot \left( -\frac{44}{55} \right) \)
9. \( -\frac{16}{21} \cdot \left( -\frac{15}{24} \right) \)
10. \( \frac{-75}{80} \cdot \left( -\frac{16}{25} \right) \)
11. \( -\frac{3}{7} \cdot \frac{5}{12} \cdot \left( -\frac{4}{5} \right) \)
12. \( -\frac{2}{3} \cdot \frac{1}{6} \cdot \left( -\frac{3}{4} \right) \)
13. \( -\frac{3}{5} \cdot \left( -\frac{5}{6} \right) \cdot \left( -\frac{6}{7} \right) \)
14. \( -\frac{4}{9} \cdot \left( -\frac{7}{8} \right) \cdot \left( -\frac{36}{49} \right) \)
15. \( -\frac{33}{40} \cdot \left( -\frac{17}{18} \right) \cdot \frac{8}{11} \)
16. \( -\frac{11}{36} \cdot \left( -\frac{10}{21} \right) \cdot \frac{48}{55} \)
17. \( \frac{15}{24x} \cdot \frac{8x}{5y} \)
18. \( \frac{16a}{21x} \cdot \frac{7x}{8y} \)
19. \( \frac{24ab}{25x} \cdot \frac{5xy}{8b^2} \)
20. \( \frac{15ab}{12xy} \cdot \frac{8x^2y}{5a^2b} \)

Divide the given rational numbers. Leave your answers as fractions.

21. \( \frac{8}{15} \div \frac{16}{25} \)
22. \( \frac{18}{19} \div \frac{12}{95} \)
23. \( \frac{5}{12} \div \left( -\frac{3}{4} \right) \)
24. \( \frac{16}{21} \div \left( -\frac{4}{7} \right) \)
25. \( -\frac{2}{9} \div \left( -\frac{11}{15} \right) \)
26. \( -\frac{5}{8} \div \left( -\frac{1}{4} \right) \)
27. \( -\frac{4}{9} \div \frac{8}{9} \)
28. \( -\frac{7}{10} \div \frac{4}{5} \)
29. \( \frac{7}{8} + 14 \)
30. \( \frac{5}{6} + 15 \)
31. \( \frac{3xy}{5a^2b} + \frac{9xy^2}{10ab^2} \)
32. \( \frac{4y^2}{5ab} + \frac{8x^2y^2}{15ab^2} \)
33. \( \frac{15x^3y^2}{24a^2b} + \frac{5x^2y^3}{6ab^3} \)
34. \( \frac{3xy^2}{4ab^3} + \frac{9x^2y^2}{10a^2b^3} \)

Perform the following multiplications and divisions. Write all answers as mixed numbers.

35. \( 13 \frac{1}{3} \cdot 5 \)
36. \( 12 \frac{4}{5} \cdot 8 \)
37. \( 14 \frac{1}{4} + 5 \)
38. \( 9 \frac{1}{3} + 6 \)
39. \( 3 \frac{3}{4} \cdot 1 \frac{4}{5} \)
40. \( -2 \frac{2}{3} \cdot 4 \frac{1}{6} \)
41. \( -5 \frac{3}{4} + 6 \frac{2}{3} \)
42. \( -9 \frac{2}{3} + \left( -3 \frac{1}{6} \right) \)
43. \( -8 \cdot 3 \frac{3}{4} \)
44. \( -12 \cdot 6 \frac{2}{3} \)
45. \( 4 \frac{1}{4} \cdot \left( -2 \frac{1}{2} \right) \)
46. \( 3 \frac{1}{3} \cdot \left( -2 \frac{2}{5} \right) \)
47. \( -3 \frac{3}{4} \cdot \left( -2 \frac{4}{5} \right) \)
48. \( -6 \frac{3}{4} \cdot \left( -3 \frac{5}{9} \right) \)
49. \( -6 \frac{1}{4} + \frac{5}{6} \)
50. \( -5 \frac{1}{4} + \frac{4}{5} \)
51. \( -8 \frac{3}{4} \cdot \frac{1}{5} \)
52. \( -5 \frac{3}{5} \cdot \frac{1}{7} \)
53. \( -5 \frac{1}{4} + \left( -2 \frac{2}{3} \right) \)
54. \( -3 \frac{3}{4} + \left( -2 \frac{4}{5} \right) \)

Multiply the following decimals.

55. \( 0.52 \times 3.6 \)
56. \( 0.65 \times 8.4 \)
57. \( -0.06 \times 245.8 \)
58. \( -0.09 \times 338.7 \)
59. \( 6.4 \times (-24.85) \)
60. \( 9.6 \times (-13.42) \)
61. \( -14.03 \times (-0.7) \)
62. \( -22.91 \times (-6.5) \)
Divide the following decimals. If a decimal is repeating, be sure to carry enough divisions so that a pattern is indicated.

\[ 63. \quad 0.000005683 \times 1000 \quad 64. \quad 0.000437 \times 1000 \]
\[ 65. \quad -0.000369 \times 10000 \quad 66. \quad -0.00525 \times 10000 \]

Compute each of the following whole number divisions using (a) remainders, (b) mixed numbers, and (c) decimals. If a decimal is repeating, be sure to carry enough divisions so that a pattern is indicated.

\[ 67. \quad 8.04 \div 12 \quad 68. \quad 9.63 \div 15 \]
\[ 69. \quad -5.043 \div 0.03 \quad 70. \quad -9.642 \div 0.04 \]
\[ 71. \quad 3.52 \div (-0.4) \quad 72. \quad 4.125 \div (-1.25) \]
\[ 73. \quad -13.584 \div (-0.12) \quad 74. \quad -26.85 \div (-0.015) \]
\[ 75. \quad 1.553 \div 0.033 \quad 76. \quad 9.865 \div 0.024 \]
\[ 77. \quad 95.68 \div 100 \quad 78. \quad 103.5 \div 1000 \]
\[ 79. \quad 5.683 \div 1000 \quad 80. \quad 2.37 \div 10000 \]
\[ 81. \quad 0.468 \div 100 \quad 82. \quad 0.0985 \div 1000 \]

Answer each of the following application questions. Be sure to read the question, interpret the problem mathematically, solve the problem, then answer the question. You should answer the question in the form of a sentence.

\[ 93. \quad \text{Carolyn buys a car for which she makes car payments of } \$243.56 \text{ per month for } 5 \text{ years. What is the total amount she pays for the car?} \]
\[ 94. \quad \text{Todd leases a car for which he makes lease payments of } \$698.94 \text{ per month for } 2 \text{ years. What is the total amount of his lease?} \]
\[ 95. \quad \text{A small company pays } \$56.87 \text{ per week for advertising in the local paper. What is the total cost of advertising for the company for one year?} \]
\[ 96. \quad \text{Linda pays } \$24.86 \text{ per week for some rent-to-own furniture. How much does she pay for the furniture for one year?} \]
\[ 97. \quad \text{Deborah can get pencils at an office supply store for } \$1.15 \text{ each. How many can she buy for } \$41.40 \text{? (Assume she does not have to pay tax.)} \]
\[ 98. \quad \text{Alfred pays } \$1.75 \text{ per day for his bus commute in San Francisco. If he budgets } \$591.50 \text{ for his bus commuting, how many days does he plan to commute?} \]
99. A farmer sold \( \frac{3}{5} \) of his \( 56 \frac{1}{2} \) tons of hay. How many tons of hay did he sell?

100. Jerry sold \( \frac{3}{4} \) of his \( 120 \frac{2}{3} \) tons of walnuts. How many tons of walnuts did he sell?

101. The value of land in a small town is $12,500 per acre. A small parcel of a lot is to be purchased by the town for road improvement. If the size of that parcel is \( \frac{3}{40} \) acre, how much should the town expect to pay the owner for it?

102. Ross owns a lot which is \( 1 \frac{1}{3} \) acres in size. If the value of land in his area is $48,000 per acre, what is the value of his lot?

103. Hank buys a snowmobile for $3180, and pays for it with a two year interest-free loan. If he makes equal monthly payments, how much are his payments?

104. Sasha borrows $13,707 from her niece to buy a car. Her niece charges no interest, but requires equal monthly payments for 5 years. How much are Sasha’s monthly payments?

105. Don leaves \( \frac{1}{6} \) of his estate to each of his six children. If each child inherits $12,846, how much was the total estate?

106. Martha leaves \( \frac{4}{15} \) of the value of her vineyard to a charity. If the charity inherits a value of $68,452, what is the total value of her vineyard?

107. Todd claims \( \frac{3}{8} \) of his cellular phone bill as a tax write-off for business expenses. If his tax write-off was $726 last year, what was his total cellular phone bill?

108. Mary claims \( \frac{5}{8} \) of her household expenses as a tax write-off for the care of a disabled person. If her tax write-off was $5,840 last year, what were her total household expenses?

109. John owns 120 shares of a stock which drops \( 14 \frac{3}{8} \) per share in one day. How much money does he lose in that day?

110. Dennis owns 480 shares of a stock which raises \( 5 \frac{3}{4} \) per share in one day. How much money does he gain in that day?
Answer the following questions.

111. What number must be multiplied by $15\frac{2}{3}$ so that the product is $56\frac{1}{2}$?

112. What number must be multiplied by $12\frac{2}{3}$ so that the product is 56?

113. What number must be divided by $28\frac{3}{5}$ so that the quotient is 35?

114. What number must be divided by $15\frac{3}{4}$ so that the quotient is 60?

115. If $\frac{1}{2}$ of $\frac{3}{4}$ of a number is 60, what is the number?

116. If $\frac{2}{3}$ of $\frac{4}{5}$ of a number is 120, what is the number?