2.2 Addition of Integers

Using the number line, we are now in a position to define **addition of integers**. Suppose we wanted to add $-5 + 3$. Starting at the origin, begin by moving to $-5$ on the number line. To add 3, we move in the positive direction (to the right) 3 units, resulting in the number $-2$. Note the following figure:

![Number Line Diagram](image1.png)

**Example 1** Add the two integers: $3 + (-6)$

**Solution** Starting at the origin, move to 3 on the number line, then move 6 units in the negative direction. The final location is $-3$, so $3 + (-6) = -3$.

![Number Line Diagram](image2.png)

**Example 2** Add the two integers: $-1 + (-4)$

**Solution** Starting at the origin, move to $-1$ on the number line, then move 4 units in the negative direction. The final location is $-5$, so $-1 + (-4) = -5$.

![Number Line Diagram](image3.png)
Notice when two numbers are the same sign (as in Example 2), then addition can be done mentally. Simply add the two absolute values \((1 + 4 = 5)\), then maintain the sign of the numbers. For this reason, when adding more than two numbers, the easiest approach is to use the commutative and associative properties of addition to rewrite the problem where adding like signs is first done. Recall these properties from Chapter 1:

<table>
<thead>
<tr>
<th>Property Name</th>
<th>Property</th>
<th>Example</th>
</tr>
</thead>
<tbody>
<tr>
<td>Commutative property</td>
<td>(a + b = b + a)</td>
<td>(-3 + 6 = 6 + (-3))</td>
</tr>
<tr>
<td>Associative property</td>
<td>(a + (b + c) = (a + b) + c)</td>
<td>(6 + ((-3) + (-2)) = (6 + (-3)) + (-2))</td>
</tr>
<tr>
<td>Identity property</td>
<td>(a + 0 = 0 + a = a)</td>
<td>(-5 + 0 = 0 + (-5) = -5)</td>
</tr>
</tbody>
</table>

**Example 3** Add the integers: \(-4 + 3 + (-2) + (-1) + 4 + 2 + (-6)\)

**Solution** Adding integers of like signs first:
\[-4 + (-2) + (-1) + (-6) = -13\]
\[3 + 4 + 2 = 9\]
Starting at 0, move to \(-13\) on the number line, then move to the right 9 units. The final location is \(-4\), so \(-4 + 3 + (-2) + (-1) + 4 + 2 + (-6) = -4\).

Note: This method of adding numbers of like signs first is similar to that employed in bank statements, which often list all credits (positive numbers) and all debits (negative numbers) separately, then the final balance is computed.

Addition of integers will often be used in algebra. In algebra, an **equation** is a statement which is either true or false, depending on what number is substituted for the variable. The **solution** to an equation is the number which, when substituted, results in a true statement. For example, the statement \(x + 5 = 2\) is an equation, since it is either true or false depending on the number being substituted for \(x\). If \(x = -3\), the statement \(-3 + 5 = 2\) is true, so \(-3\) is the solution to the equation. You can verify for yourself that other values substituted for \(x\) result in a false statement.
Example 4  Determine whether or not the given integer value is a solution to the equation.

a. \( x + 4 = 2; \ x = -2 \)
b. \( x + 3 = -5; \ x = -2 \)
c. \( y + 3 = -5; \ y = -8 \)
d. \( s + 5 = 0; \ s = -5 \)

Solution

a. Substitute \( x = -2 \) into the equation and use the number line for the addition:
   \[-2 + 4 = 2 \]
   \[2 = 2 \]
   Since this statement is true, \( x = -2 \) is a solution to the equation \( x + 4 = 2 \).

b. Substitute \( x = -2 \) into the equation and use the number line for the addition:
   \[-2 + 3 = -5 \]
   \[1 = -5 \]
   Since this statement is false, \( x = -2 \) is not a solution to the equation \( x + 3 = -5 \).

c. Substitute \( y = -8 \) into the equation and use the number line for the addition:
   \[-8 + 3 = -5 \]
   \[-5 = -5 \]
   Since this statement is true, \( y = -8 \) is a solution to the equation \( y + 3 = -5 \).

d. Substitute \( s = -5 \) into the equation and use the number line for the addition:
   \[-5 + 5 = 0 \]
   \[0 = 0 \]
   Since this statement is true, \( s = -5 \) is a solution to the equation \( s + 5 = 0 \).

Note that the label we use for the variable (\( x, y, s \), etc.) makes no difference in determining the solution.

Sequences of numbers are often used in elementary schoolbooks as a way to develop patterning with numbers. The numbers in a sequence are called terms. One particular type of sequence is called an arithmetic sequence, where each term is the result of adding the same quantity (called the common difference) to the previous term. For example, the sequence 2, 5, 8, 11, 14, … is an arithmetic sequence, since 3 is added onto each term to form the successive term. The next term of this sequence would be \( 14 + 3 = 17 \).
Example 5  Find the next term in each arithmetic sequence.

a.  2, 6, 10, 14, …

b.  −5, −3, −1, 1, …

c.  6, 3, 0, −3, …

d.  −1, −6, −11, −16, …

Solution

a.  Note that 2 + 4 = 6, 6 + 4 = 10, and 10 + 4 = 14, so the common difference is 4. The next term is 14 + 4 = 18.

b.  Note that −5 + 2 = −3, −3 + 2 = −1, and −1 + 2 = 1, so the common difference is 2. The next term is 1 + 2 = 3.

c.  Note that 6 + (−3) = 3, 3 + (−3) = 0, and 0 + (−3) = −3, so the common difference is −3. The next term is −3 + (−3) = −6.

d.  Note that −1 + (−5) = −6, −6 + (−5) = −11, and −11 + (−5) = −16, so the common difference is −5. The next term is −16 + (−5) = −21.

After defining subtraction, we will again visit arithmetic sequences with another approach to finding the common difference (rather than just guessing as we did in this example).

Terminology

addition of integers
solution
common difference
equation
arithmetic sequence

Exercise Set 2.2

Find the sum of the two integers. Remember to draw number lines whenever necessary.

1.  −8 + 4
2.  −12 + 7
3.  −4 + (−3)
4.  −16 + (−7)
5.  5 + (−9)
6.  4 + (−13)
7.  19 + (−12)
8.  24 + (−15)
9.  −24 + 32
10.  −48 + 57
11.  −||8|| + (−4)
12.  −||−14|| + (−6)
13.  −||5|| + (−12)
14.  −||8|| + (−23)
15.  −||−19|| + 7
16.  −||−23|| + 16
17. \(-(-4) + 7\)  
19. \(-(-14) + (-5)\)  
18. \(-(-12) + 4\)  
20. \(-(-6) + (-23)\)

Find the indicated sums. Remember you can rearrange numbers in a more convenient order, if desired.

21. \(12 + (-4) + (-15) + 8\)  
22. \(15 + (-12) + (-14) + 7\)  
23. \(-12 + (-3) + 4 + (-7)\)  
24. \(-15 + (-6) + 8 + (-6)\)  
25. \(5 + (-23) + 17 + (-48)\)  
26. \(13 + (-42) + 26 + (-8)\)  
27. \(-(-6) + |-16| + (-16)\)  
28. \(-(-4) + |-25| + (-41)\)  
29. \(-|37| + |-9| + (-8) + 3\)  
30. \(-|16| + |-12| + (-15) + 6\)  
31. \(-|-15| + |-7| + (-9) + 13\)  
32. \(-|-62| + |-14| + (-15) + 24\)

Determine whether or not the given integer value is a solution to the equation.

33. \(x + 4 = 15; x = 9\)  
34. \(x + 4 = 15; x = 11\)  
35. \(x + 7 = 13; x = 6\)  
36. \(x + 7 = 13; x = -6\)  
37. \(y + 5 = 1; y = -4\)  
38. \(y + 5 = 1; y = 4\)  
39. \(a + 2 = -6; a = -10\)  
40. \(a + 2 = -6; a = -8\)  
41. \(t + (-3) = -5; t = -2\)  
42. \(t + (-3) = -5; t = -8\)  
43. \(w + (-12) = -4; w = -8\)  
44. \(w + (-12) = -4; w = 8\)

Find the next term in each arithmetic sequence.

45. \(2, 4, 6, 8, \ldots\)  
46. \(3, 6, 9, 12, \ldots\)  
47. \(1, 4, 7, 10, \ldots\)  
48. \(2, 5, 8, 11, \ldots\)  
49. \(-5, -2, 1, 4, \ldots\)  
50. \(-10, -6, -2, 2, \ldots\)  
51. \(8, 5, 2, -1, \ldots\)  
52. \(6, 2, -2, -6, \ldots\)  
53. \(-1, -4, -7, -10, \ldots\)  
54. \(-2, -5, -8, -11, \ldots\)  
55. \(-3, -7, -11, -15, \ldots\)  
56. \(-4, -9, -14, -19, \ldots\)
Determine whether each statement is true or false. If it is false, give a specific example which shows it is false. If it is true, explain why.

57. If \( x > 0 \) and \( y > 0 \), then \( x + y > 0 \).
58. If \( x < 0 \) and \( y < 0 \), then \( x + y < 0 \).
59. If \( x > 0 \) and \( y < 0 \), then \( x + y > 0 \).
60. If \( x > 0 \) and \( y < 0 \), then \( x + y < 0 \).
61. If \( x < 0 \) and \( y > 0 \), then \( x + y < 0 \).
62. If \( x < 0 \) and \( y > 0 \), then \( x + y > 0 \).
63. If the sum of two numbers is positive, then at least one of the numbers must be positive.
64. If the sum of two numbers is negative, then at least one of the numbers must be negative.
65. If the sum of two numbers is negative, then both of the numbers must be negative.
66. If the sum of two numbers is positive, then both of the numbers must be positive.
67. If \( x > 0 \) and \( y > 0 \), then \( |x + y| = x + y \).
68. If \( x < 0 \) and \( y < 0 \), then \( |x + y| = x + y \).
69. If \( x < 0 \) and \( y > 0 \), then \( |x + y| = x + y \).
70. If \( x > 0 \) and \( y < 0 \), then \( |x + y| = x + y \).
71. If \( x > 0 \) and \( y > 0 \), then \( |x + y| = |x| + |y| \).
72. If \( x < 0 \) and \( y < 0 \), then \( |x + y| = |x| + |y| \).
73. If \( x < 0 \) and \( y > 0 \), then \( |x + y| = |x| + |y| \).
74. If \( x > 0 \) and \( y < 0 \), then \( |x + y| = |x| + |y| \).
75. If \( x \) and \( y \) are opposites, then \( x + y = 0 \).
76. If \( x + y = 0 \), then \( x \) and \( y \) are opposites.

Solve each of the following application problems. Try to use negative numbers as much as possible, even if you can solve the problem without using negative numbers.

77. A hiker starts in the Salton Sea, at an elevation of 145 feet below sea level. He climbs up 420 feet in elevation. What is his new elevation?
78. A hiker is near Death Valley, at elevation 257. She climbs down 400 feet in elevation. What is her new elevation?
79. You wake up in Duluth, MN to a brisk –23°. During the day the temperature rises 15°, but it drops 24° during the night. What is the temperature the next morning?
80. Bob leaves to go cross-country skiing in Hibbing, MN, when the temperature is 13°. A cold front comes in, and the temperature drops 28° while he is skiing. What is the new temperature?

81. Ron calls the bank and finds that his balance is $26. He already wrote a check for $57, and he knows the bank will charge him a $15 overdraft fee. What will his new balance be?

82. Rudy checks his bank balance and finds that it is –$45. He deposits a check for $28. What will his new balance be?